# Domesticating Mathematics in the African Mother Tongue

by

Abdul Karim Bangura

*Abdul Karim Bangura* is professor of Research Methodology and Political Science at Howard University, researcher-in-residence of Abrahamic Connections and Islamic Peace Studies at the Center for Global Peace in the School of International Service at American University. He holds a PhD in Political Science, a PhD in Development Economics, a PhD in Linguistics, a PhD in Computer Science, and a PhD in Mathematics. He is the author of 75 books and more than 600 scholarly articles. One of his recent books titled *African Mathematics: From Bones to Computers* (2012) is the winner of the prestigious Cecil B. Curry Book Award for 2012. Bangura is fluent in about a dozen African languages and six European languages, and he is studying to increase his proficiency in Arabic, Hebrew, and Hieroglyphics. He is also the recipient of many teaching, scholarly and community service awards, a member of many scholarly organizations, and he has served as President and United Nations Ambassador of the Association of Third World Studies.

## Abstract

This paper is about why and how African-centered Mathematics can be a driving force in Africa's development efforts. That Africa was the center of Mathematics history for tens of thousands of years is hardly a matter of dispute. From the civilizations across the continent emerged contributions which would enrich both ancient and modern understandings of nature through Mathematics. Yet, today, scholars and other professionals working in the field of Mathematics Education in Africa have identified a plethora of problematic issues in the endeavor. In this paper, I argue that a major reason for these problems is that the African mother tongue has been greatly neglected in the teaching of Mathematics in Africa. This situation must be changed if the continent is to benefit from the tremendous opportunities Mathematics offers. While a great deal of work exists on the connections between Linguistics and Mathematics in general, few can be found on the nexus between African languages and Mathematics in particular, and these latter works are not linguistics-theoretically grounded. Thus, this essay begins by identifying the objects to the study of Linguistics and Mathematics and delineates which ones they study in common. Next, since the object of the study of Linguistics is language, the nine design features of language are employed to examine each of the objects as it pertains to African languages. After that, Mathematics of Sustainability and Mathematics of Tipping Points are suggested as a means to help Africa's development efforts.

## Introduction

> As we describe the Rules of False Position, it is critically important to realize that the Egyptians and the Babylonians did not know algebra, indeed it did not exist at that time, nor did they have the notion of an equation; hence, they could not make obvious simplifications and they did not work with a general rule (Papakonstantinou and Tapia, 2013:504).

Given the preceding quotation, it is disheartening that in the year 2013, such an ignoramus statement could be made by Joanna M. Papakonstantinou and Richard A. Tapia, two well-placed mathematicians. And if I may quote their brief biographies in *The American Mathematical Monthly* (vol. 120, no. 6, June-July 2013) in which the statement is made, "Joanna M. Papakonstantinou received her B.A., M.A., and M.A.T., as well as her Ph.D., from Rice University, under the direction of the second author. After completing her postdoctoral work at Rice University in the Computational and Applied Mathematics Department, she joined PROS Revenue Management as a Senior Associate where she served as a science expert in the Center of Excellence. Currently, she works as the Senior Science Consultant at Advanous. She remains involved in research in the field of optimization, writes curriculum and teaches, and actively participates in outreach activities" (Papakonstantinou and Tapia, 2013:517). "Richard A. Tapia, 2011 recipient of the National Medal of Science [the top award the United States government offers its researchers given to him by President Barack Obama—author's note], holds the rank of University Professor in the Rice Department of Computational and Applied Mathematics. He is also the Director of the Rice University Center for Excellence and Equity in Education. Due to Tapia's efforts, Rice has received national recognition for its educational outreach programs and has become a national leader in producing women and underrepresented minority Ph.D. recipients in the mathematical sciences" (Papakonstantinou and Tapia, 2013:517). Even more disheartening is that the editor of the journal, Scott T. Chapman, another well-placed mathematician at Sam Houston State University, the three anonymous referees, and the authors' colleague, Michael Trosset, who the authors credited "for suggestions that greatly improved the paper" (Papakonstantinou and Tapia, 2013:516), ignored the pernicious statement.

It is quite obvious that all of these mathematicians have not read Cheikh Anta Diop's famous work, *Civilization or Barbarism* (1981/1991), and serious works on African Mathematics in which it is clearly demonstrated that ancient Egypt, in fact, is the origin of Algebra. Algebraic mathematical series, simple equations, quadratic equations, balance of quantities (*pesou*) were all invented in Egypt. Even the Rules of False Position, which is the main focus of Papakonstantinou and Tapia's paper, were invented by the ancient Egyptians. Add to these the many arithmetic and geometric techniques that were invented in ancient Egypt and other parts of Africa.

Indeed, that Africa was the center of mathematics history for tens of thousands of years is hardly a matter of dispute. From the civilizations across the continent emerged contributions which would enrich both ancient and modern understanding of nature through mathematics. Yet, today, scholars and other professionals working in the field of mathematics education in Africa have identified a plethora of problematic issues in the endeavor. These issues include attitudes, curriculum development, educational change, instruction, academic achievement, standardized and other tests, performance factors, native speakers, etc. (for more on this, see Bangura, 2012). In this paper, I argue that a major reason for these problems is that the mother tongue has been greatly neglected in the teaching of mathematics in Africa. Indeed, as Mamokgethi Setati (1998, 2002, 2003, 2005a, 2005b, 2008) has demonstrated numerously in the case of South Africa, even though teachers and learners who employ African languages in mathematics education position themselves in relation to mathematics and, concomitantly, epistemological access, and use those languages as instruments of solidarity, English is the dominant language in the classroom, its ascendancy privileged by procedural mathematics discourse. This situation must be changed if Africa is to benefit from the tremendous opportunities mathematics offers.

Linguists have for a long time been concerned with the context of teaching any subject/text, be it qualitative or quantitative. Students of linguistics can readily find examples where the meaning of an utterance changes with its contextual framework. The following are examples: (a) General Motors had no success selling the Chevy Nova in Mexico because Nova (*no va*) means "no go" in Spanish; (b) the "I'm a Pepper" advertisement that was successful in the United States had to be changed in England when Dr. Pepper's management realized that *pepper* is a slang word for prostitution to the English; (c) the "Pepsi, the choice of a new generation" advertisement translated into Taiwanese got many Taiwanese angry after they flocked to buy Pepsi only to realize that their dead ancestors were not coming back to life. The extent of the problem became more widely known when computer programmers were faced with the vexing task of translating texts. A famous example was an attempt to translate "The spirit is strong but the body is weak" into Russian. The resulting phrase was "The Vodka is stale and the meat is rotten." Yet still, linguists also know that any subject, even mathematics that has become extremely abstract, can be taught effectively in any language once the appropriate tools are made available.

While a great deal of works exists on the nexus between linguistics and mathematics in general, only a small number can be found on the nexus between African languages and mathematics in particular, and these latter works are not linguistics-theoretically grounded. Thus, this essay begins by identifying the objects of study of linguistics and mathematics and delineates which ones they study in common. Next, because the object of study of linguistics is language, the nine design features of language are employed to examine each of the objects as it pertains to African languages and mathematics. The nine design features are (1) mode of communication, (2) semanticity, (3) pragmatic function, (4) interchangeability, (5) cultural transmission, (6) arbitrariness, (7) discreteness, (8) displacement, and (9) productivity.

Since to offer examples for each of these features from each of the more than 2,100 African languages would require at least a book-length manuscript, a small number of examples from a small number of languages across the continent are presented for the sake of brevity. After that, the discussion turns to the role for mathematicians in fulfilling the African Renaissance—defined by Dani Wadada Nabudere as the initiative to recapture the basic elements of African humanism (*ubuntu*, *eternal life*, and *immanent moral justice*) as the path to a new humanistic universalism. He quotes Chancellor Williams as stating that this initiative "is the spiritual and moral element, actualized in good will among men (and women), which Africa itself has preserved and can give to the world" (Nabudere, 2003:4). The proposition in this section is that just as mathematicians had played a major role in the development of African societies during antiquity (see Bangura, 2012), they are sorely needed now to work with experts in other disciplines to help fulfill the African Renaissance. In the end, conclusions are drawn and recommendations are offered.

Thus, the observations made in this paper are not directed at the discovery of any method or pedagogical panacea. They are presented in complete modesty in the belief that what matters most is not the method but the teacher. May the observations serve then, at best, as a starting point for that self-examination. Since teaching about the nexus between African languages and mathematics is one to which many of us are deeply committed, it is the hope that this paper will not only inspire colleagues to give serious consideration to its suggestions and perplexities, but also strive to suggest better solutions than those proposed.

## Objects of Study of Linguistics and Mathematics

Before identifying the objects of study of linguistics and mathematics and delineating which ones they study in common, it makes sense to begin with brief definitions of the two fields, with the caveat that, as many linguists and mathematicians have pointed out, providing concise and meaningful definitions of these fields on which everyone can agree is virtually impossible. Nonetheless, to avoid providing working definitions of the fields will cause more havoc than it solves. Linguistics, according to Francis Dinneen (1995), can be generally defined as the scientific study of *language*. Mathematics, following Keith Devlin (2003), can be generally defined as the systematic study of *change*, *quantity*, *relation*, *space*, *structure*, and other topics dealing with *entity*, *form*, and *pattern*. Consequently, these italicized aspects of the definitions constitute the objects of study for the two disciplines, whether they are dealing with numbers, other fields, hypothetical abstractions, or other things.

With this backdrop of what linguistics and mathematics are and their objects of study, the question to be probed here then should be about those objects the two fields study in common. The answer must show that the two disciplines are parallel and must interact where they happen to coincide, especially since there is even a sub-discipline in linguistics called mathematical linguistics.

The most common answer one usually finds in many textbooks and articles is that mathematics is a universal language among the numerous different languages spoken all around the world and that these languages are constantly changing just as mathematics; thus, it is only natural that some linguists use the tools of their discipline to study mathematics. Scholars specializing in mathematical linguistics, however, go further than that. For example, Geoffrey Pullum and András Kornai (2003) provide a definition of mathematical linguistics and then survey the following traditional subfields of linguistics and how they study and employ mathematics: phonetics, phonology and morphology, syntax, semantics, and model-theoretic syntax. The following is a synopsis of their discussion.

Pullum and Kornia define mathematical linguistics as the reverie of mathematical structure and methods that are relevant to linguistics. They note that the agency of the empirical subject matter of mathematical linguistics is relatively indirect as it is other subareas of applied mathematics, since theorems are proved not for their applicability but for their mathematical value. Nonetheless, they add, the internal subdivisions of mathematical linguistics are still best guided by the internal organization of linguistics (2003:17).

For phonetics, Pullum and Kornai state that Hidden Markov Models (HMMs) are the key structures of both phonetics and mathematics. They note that the way the structure of HMMs is set up as a "discrete, psychologically relevant underlying units as hidden states coupled with conditions, physically relevant output" makes the models important to the study of phonetics and mathematics (2003:17). They add that if a modality is changed from spoken to written language, HMMs will be applicable because the models assume a Markovian underlying structure which is ideal for modeling the succession of linguistic units (2003:17).

As it pertains to phonology and morphology, Pullum and Kornai point out that Bloomfieldian postulates provide the fundamental apparatuses of mathematical linguistics, particularly the notion of hierarchical structures made up of relatively stable recurrent items developed principally on the basis of phonological and morphological attributes. They mention that the three Chomskian theoretical models—finite state automata (FSA), context-free grammars (CFGs), and context-sensitive grammars (CSGs)—and/or the even more powerful unrestricted rewriting systems (URSs) are critical for describing linguistic structure and emerged as the basis for work on formal language structure theory within computer science. They note that finite state tranducers (FSTs), which are very useful for mathematical work on phonology, are employed to describe the definitive formalization of theoretical phonology and morphology that was proposed by Noam Chomsky and Morris Halle (1968). Pullum and Kornai add that given the near-phonetic nature of Russian orthography, HMMs contributed to phonology and statistical linguistics. They conclude that "today the mathematical apparatus of phonology and morphology is centered on the study of deterministic, nondeterministic, and probabilistic FSTs" (2003:17-18).

In terms of syntax, Pullum and Kornai state that Noam Chomsky's formalization of immediate constituent analysis utilizing context-free grammars (CFGs) was his first significant technical contribution to the field of linguistics. They note that to this day, CFGs maintain a key position in the design of programming languages, even though some of the widely-used programming languages transcend CFGs in some ways. They add that compiler designs emerged as the home of much of the early work in mathematical linguistics concerned with developing efficient parsing techniques (2003:18).

In the case of semantics, Pullum and Kornai inform us that there was insistence during early efforts dealing with the meaning of expressions in linguistic semantics to provide translations of those expressions in a representational system of some sorts within generative grammar. They point out that while philosophers objected that no such representation in any vocabulary can yield a specification of meaning, the work of Richard Montague (1974) led to providing natural language expressions with actual model-theoretic interpretations just as is done with formal languages in logic. Nonetheless, Pullum and Kornai, argue, "On the whole, approaches to semantics based on information theory are still largely restricted to lexical semantics, though many tasks such as machine translation that were originally believed to require sophisticated semantic analysis are now often performed by purely statistical models" (2003:18-19).

Relating to model-theoretic syntax, Pullum and Kornai state that logical theory with well-formed structures in the language as its models have emerged as the recent line of research that connects model theory to syntax. They point out that CFG is used to devise monadic second-order logic that characterizes the sets of tress that can be generated. They also mention that phrase structure rules are employed to make directly the set of trees CFGs make, and that modal logic is utilized to extend these notions to structures that are more complex than trees (2003:18-19).

## Mode of Communication

Sending and receiving messages define the very nature of a system of communication. The means by which these messages are transmitted and received is refereed to as *mode of communication*. Users of human languages employ voice (or oral language) and gesture (or body language) to transmit messages. These two modes of communication are imperative for the transmittal of the complex sorts of messages that languages require (Bergmann et al., 2007:17).

As Ekkehard Wolff reminds us, like other cultures anywhere else in the world, African cultures have developed specific patterns of language usage for effective communication. Consequently, a scale of acceptance at a given instance of speaking exists on the continent. Wolff cites an example from the Itsekiri and Okpe in the Delta region of Nigeria who use different terms for such words as 'blood,' 'fire,' and 'firewood' according to whether the wood is being used during day time or at night.

He adds that terms of avoidance, or taboo, and respect are among the best-known instances of culturally determined language usage for effective communication in Africa. For this mode of communication, he cites the example of the Xhosa and Zulu in Southern Africa who refer to it as *hlonipa* (Wolff, 2000:305).

In his study of meanings in Madagascar, Øyvind Dahl shows the ways language is employed in communication to reduce uncertainty, act effectively, and defend or strengthen the ego. He notes that while in some occasions words are used to ward off anxiety, in other occasions they are employed to evolve more deeply satisfying manners of expression. Thus, according to him, the aim of communication is to increase the number and consistence of a person's "meanings within the limits set by patterns of evaluation that have proven successful in the past," the "emerging needs and drives, and the demands of the physical and social setting of the moment" (Dahl, 1999:9).

In her discussion of spoken and gesture counting in Africa, Claudia Zaslavsky provides a brief description of the communicative act and provides numerous examples across the continent. She states that gestures are employed to either replace the spoken word or to emphasize a point. She notes that gesture may be used to circumvent the taboo of counting living things and to overcome language barriers between people of different backgrounds. She adds that in certain contexts, fingers are utilized by both Western educated and uneducated individuals as an aid in mental arithmetic (1999:238). The following is an example of spoken and gesture counting from the Shambaa language of northeast Tanzania (Zaslavsky, 1999:238-239):

6 = *mutandatu* = *ntatu na ntatu* = 3 + 3.
7 = *mufungate* = *funga ntatu* = bind 3 (fingers); from the ten
fingers seven remain; therefore a subtractive formation.
8 = *munane* = *ne na ne* = 4 + 4.
9 = *kenda*, an alien word, used here, as in other languages
without the class prefix.

## Semanticity

*Semanticity* calls for all signals of communication to possess a meaning and function. It is vital that a speaker/writer and a hearer/reader share a similar idea about what is being talked or written in order to have successful linguistic communication. Even when a person hears a word s/he does not know, s/he nevertheless assumes that it must possess some meaning (Bergmann et al., 2007:17-18).

Funwi Ayuninjam informs us that in the Mbili language of Cameroon, there exists significant crisscrossing between the prefixes and the semantic categories; in some cases, there is crisscrossing even within a noun class—a cross-categorization that is rather common in Bantu languages. He notes that nouns and verbs in Mbili possess independent categorical semantic properties, thereby being classified as two distinct categories, albeit they are related in the sense that they share syntagmatic relations by which they are linked by forms. He adds that nouns and adjectives derive from verbs, but not vice versa, based on valency of range of usage, morphological dependence, semantic dependence, and semantic range (1998:201-202). Furthermore, in terms of semantic features, Ayuninjam mentions that there are verbs which express aspectual meanings but do not have overt grammatical markers, and that selectional restrictions in these features are analogous to collocational restrictions, although some are semantic and others are formal (1998:334-335).

According to Denis Creissels, African "languages that have gender generally have a morphologically complex form of plural marking, characterized by a fusion of gender markers and number markers, and variations in gender and number manifest themselves through morphemes affixed to the head noun and to (some of) its modifiers, in an agreement relationship. In these languages, plural marking tends to function on a 'semantic' basis, which means that plural markers tend to be present in every noun phrase referring to a plurality of individuals, irrespective of their communicative relevance" (2000:246-247). He cites the case of Hausa whose plurals divide into six basic types, most of them having two or more subtypes, and a few that are entirely irregular plurals. He adds that plural formation in Hausa involves both suffixes and vowel insertion (e.g., *gàrema:* pl. *garèmani:* 'large hoe'), consonant reduplication (e.g., *ko:fa:* pl. *ko:fo:fi:* 'doorway'), and change in tone (2000:247).

John Watters mentions that in Central Sudanic languages such as Ma'di and Western Nilotic, the perfective requires subject-verb-object (SVO) and the imperfective requires a subject-object-verb (SOV) word order. He points out that most often in these languages, the first object has a benefactive semantic role (i.e. the one who benefits from the action), which is often referred to as the indirect object' (2000:199).

Problem 64 of the *Rhind Papyrus* dealing with a distribution of differences is an arithmetic progression that possesses a meaning and function. According to Cheikh Anta Diop, the problem "consists of dividing ten loaves of bread among ten persons, in such a manner that the difference between two consecutive persons is one-eighth of a *hekat*. One obtains the same result as the scribe applying the classical formula of an arithmetic progression" (1981/1991:270):

$$l = a + (n - 1)d$$

where $l$ = the last term, $a$ = the first term, and $d$ = the common difference: $^1/_8$.

## Pragmatic Function

A communication system is said to have a *pragmatic function* when it serves some useful purpose. Such functions include language used to help individuals to stay alive, to influence others' behaviors, to find out more about the universe, to ask for food, to yell for help, and to influence voting behavior. Some communicative acts such as gossip are usually questioned, but they do have a pragmatic function in helping us to understand our social environment (Bergmann et al., 2007:18).

In my book on the African National Anthem *Nkosi Sikelel' iAfrika* (2011), I provide a pragmatic linguistic analysis of the song by delineating its deixis, presuppositions, implicatures, and speech acts. The South African Enoch Mankayi Sontaga composed what later became the African National Anthem in 1897. This song was composed at a time when Africans in South Africa were living in a period of high political expectation. It is a product of the politico-religious movement of the time, which took the form of the religion of the oppressed, and became the ideological expression of the progressive tendencies of the anti-colonial resistance (Meli, 1988:32, Bangura, 2011:1).

In his examination of Malagasy, Dahl provides many examples of the linguistic pragmatics of communication in the language, including communicative style, cultural values, etc. He examines the parameters upon which the society widely relies, such as "truth," "honor," "non-confrontation," "self-assertion," and "directness" (1999:125-125). He also discusses the some linguistic pragmatic and structural aspects of the use of the language by focusing on a special case of communication in public decision-making. He concludes that "In a society that values commonality as much as the Malagasy society, the procedures for argumentation, decision making, and choice of speech mode have great influence on how the partners of communication decode each other's message" (1999:143).

In examining the use of plural markers in African languages that largely correlate both with the morphological nature of plural markers and with the presence versus absence of a gender system, Creissels finds that "languages devoid of a gender system generally have a single plural marker with the morphological status of a plural affix, and such plural markers tend to be used on a 'pragmatic' basis, that is to be employed only when plurality is both communicatively relevant and not implied by the context, at least in the case of nouns that do not refer to persons" (2000:246). He cites the Eastern Sudanic branch of Nilo-Saharan and all branches of the Afroasiatic phylum as possessing extreme cases of morphologically complex plural marking (2000:246).

In the 1970s, during excavations near what is today called Border Cave in the Lebombo Mountains between South Africa and Swaziland, a small piece of the fibula of a baboon (dated approximately 35000 BC) was found marked with 29 clearly defined notches. At more than 37,000 years old, the bone, which resembles the calendar sticks still in use by the San people in Namibia, ranks with the oldest mathematical objects known (www.math.buffalo.edu/mad/Ancient-Africa). It has also been suggested that since the Lebombo Bone was used as a lunar phase counter, then women may have been the first mathematicians, as keeping track of menstrual cycles requires a lunar calendar (*The Internet Encyclopedia of Science* 2007).

As Richard Mankiewicz asserts (2001), the Lebombo Bone bears witness to the existence of a very sophisticated accounting system which enabled humans to master time, and it is the first visible hint of the emergence of calculation in human history. The Lebombo Bone has up to six phases, suggesting that it represents a binary calendar. In the words of Gnaedinger,

> The pattern consists of 14 by 14 dots, linked by two more dots. The 29 dots of the top line can be read as a lunar calendar. There are 30 spaces between and next to the 29 dots. Read the spaces and dots as follows: 30 spaces plus 29 dots plus 30 spaces plus 29 dots plus 30 spaces …, yielding 30 29 30 29 30 29 30 29 30 29 30 29 30 29 30 … nights or 30 59 89 118 148 177 207 236 266 295 325 354 384 413 443 472 502 … nights for 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 … lunations. There are 2 x 13 x 14 dots below the top line, yielding 364 days; add the dot in the middle of the bottom line and you obtain 365 days for a year (Gnaedinger, 2005).

In essence, the Lebombo bone exemplified a communication system with a *pragmatic function* because it served some very useful purpose.

## Interchangeability

When individuals are able to both transmit and receive messages, they can engage in *interchangeability*. An individual can both produce messages by gesturing, speaking, or writing and understand the messages of others by looking, listening, or reading (Bergmann et al., 2007:18).

The fact that there are many indigenous African writing systems, including Adinkra, Afan-Oromo, Akan, Amharic, Bassa, Egyptian, Ethiopic, Mende, Meroitic, Nsibidi, Rock Arts, Tifingh, Vai, and Wabuti, and that foreign writing systems have been employed to develop orthographies for most and could be used to develop the same for the remaining languages, is proof that interchangeability is a significant feature in African languages.

In fact, as Diop reminds us in his book titled *Civilization or Barbarism: An Authentic Anthropology*, “it is a typically Negro African language that has been the oldest written language in the history of humanity. It began 5,300 years ago, in Egypt; whereas the most ancient testimony to an Indo-European language (Hittite) goes back to the XVIIIth Egyptian Dynasty (1470 BC) and this, probably under the influence of the political and cultural domination of Asia Minor by Egypt” (1981/1991). As Diop also points out in one of his other books, *Precolonial Black Africa*, Cameroon has a hieroglyphic script, the systematic development of which by the Ndyuya may be of contemporary date, but its origins are quite old. He mentions that the syllabic script of the Vai in Sierra Leone and the cursive script of the Bassa have been well investigated, and in Sierra Leone these scripts have been used to write modern texts (1987:185).

About the ancient Manding script, Clyde-Ahmad Winters informs us that it dates back to about 7,000 years BP (Before the Present) and was used by the Mande to write or engrave inscriptions throughout the Western Sahara. It was a syllabic script similar to the Vai script. These scripts were developed by the demands of long distance trade. Merchants needed them to keep record of their business transactions, and they were later used to preserve religious doctrines and writing obituaries. Among the Mande, the *Kuma*, which means the word, is considered sacred. Thus, written amulets have long been recognized as containing magical power (Winters, 1986:208-209).

Through his research, Winters discovers that the Mande script has around 200-350 signs and around 40 different forms. The inscriptions are read from right to left or top to bottom. Due to the high frequency of *pf* disyllabic roots, of the kind CVCV (consonant/vowel/consonant/vowel), CVN (consonant/vowel/nasal), or CVV (consonant/vowel/vowel), Mande was written in syllabary. The reduction of disyllabic roots resulted into the monosyllabic roots of CV. The same sign can be used to represent different phonetic sounds; therefore, several characters represent different phonetic values. Six vowels are used in the syllabary: a, e, é, è, o, and ò. The most common syllabic form is the monosyllabic. While compound nouns are not much in use in the script, derivative nouns and adjectives are formed by suffixes. The major suffixes used are those of possession, or nationality, which are joined to names or serve to form names and verbs. Personal pronouns are all written in the second and third person singular. The most common verbs are the attributive. The major nouns deal with death, burial customs, agriculture, and talismans (Winters, 1986:211-212).

The following anecdote from Zaslavsky is a good example of interchangeability in African mathematical practices through gesture:

> David Zarembka, an American volunteer teacher, lived and worked among the refugees from Rwanda in western Tanzania. At the market one day he asked the price of an item. "One shilling," replied the merchant, with a wave of his hand. David offered the shilling, but the merchant refused to sell the article. Again he waved his hand, with two fingers outstretched. At last David understood that the price was one shilling, twenty cents. The people of Rwanda accustomed to the Belgian franc, convert ten cents in shilling and two francs, indicated by waving two fingers (1999:238).

This example shows how gestures are employed to replace the spoken word when exchanging messages. They are also be used to emphasize a point during communication, as stated earlier.

## Cultural Transmission

*Cultural transmission* refers to the fact that there are aspects of language which people can acquire only through communicative interactions with other users of that language system. Even children, whose ability to learn languages may seem innate, must still learn all of the specific signals of those languages by interacting with other speakers. The language we acquire as children is not influenced whatsoever by our genetic or hereditary background (Bergmann et al., 2007:18).

Richard Hayward (2000) informs us that linguists should be particularly aware of the intellectual debt owed by the Western world to speakers of Afroasiatic languages because of the enormous breakthrough they attained in the expression of language in written form and made possible alphabetic writing. This breakthrough has facilitated the transmission of cultural beliefs and practices in every human endeavor.

According to Christopher Ehret, through most of human history, as long as there was a society to which a language belonged, that language would be used as a medium of social and cultural communication. He also points out that the language can die as people lose the sense that they belong to a community that is distinct from those of other communities and refuse to pass the language down to the younger generations. He cites the case of the Dahalo, a language of the Southern Cushitic subgroup of the Afroasian language family spoken in Kenya. He states that more than 2,000 years ago, the Dahalo who were gatherer-hunters spoke their own language. He adds that after periods of close relations with their neighboring, dominant Southern Cushitic-speaking family society, the Dahalo gave up using their Khoisan language. He also mentions that the Dahalo managed to keep many of the words from their old language and continue their older food-collecting way of life (2000:275-276).

Dahl provides six respective cultural "frames of reference" in Malagasy. The first frame of reference is that truth is expressed in social relations, not in the accuracy of propositions. Thus, the Malagasy word for truth is *marina*, whose root word is *arina*, whose literal meaning is to express something plain, without ups and downs. The second frame of reference is to avoid open confrontation. The third frame of reference is for an individual to adapt him/herself to circumstances. The fourth frame of reference is for a person to strive for harmony (*mampitovy tantana*), balance (*mandanjalanja*), and the golden middle (i.e. avoiding confrontation). The fifth frame of reference is that one must not express his/her feelings openly. And the final frame of reference is that self-praise is bad (Dahl, 2000:126-127).

African cultural transmissions can be found in American English, as Joseph Holloway and Winifred Vass demonstrate in their book, *The African Heritage of American English*. They identify eight African culture clusters in South Carolina alone: Akan, Mande, Mano River, Niger Delta, Bakongo, Ovimbundu, Yao, and Unknown (1993:xxvii). This outcome is due 'largely' to the Trans-Atlantic Slave Trade which took place from the $16^{th}$ to the $19^{th}$ Century and saw more than 12 million enslaved Africans brought to the Americas and the Caribbean. I say 'largely' because there is an abundant amount of evidence which proves that Africans were already in the New World almost 200 years before Christopher Columbus' voyage to the area, as King Abu Bakri/Bakar of the Malian Empire had spearheaded a series of voyages to the area in 1310 (DawaNet, 2013).

In his book, *Geometry from Africa: Mathematical and Educational Explorations*, Paulus Gerdes tells us that while the *sona* sand tradition was cultivated primarily by the Chokwe, related peoples such as the Ngangela and Luchazi also did so. These peoples inhabit eastern Angola, neighboring zones of Congo/Zaire, and northwest Zambia. Gerdes provides numerous examples of how culture is transmitted through the *sona* sand tradition that is undergirded by mathematics. An excellent example is the monolinear "chased chicken" design of dimensions 5 X 6, 9 X 10, and 3 X 8, respectively. When the same algorithm is applied to a grid of dimension 10 X 5, the three "chased chicken" lines are needed to embrace all points on the grid (Gerdes, 1999:182). Gerdes suggests that one can then ask about how the number of "chased chicken" lines are related to the dimensions of a rectangular grid: [$g(r, c) = ?$]. One can now conjecture the following, according to Gerdes (1999:184):

$G(r, c)$ is a common divisor of $\frac{r+1}{2}$ and $\frac{c+2}{2}$

or still further

$f(2m, 2n + 1)$ is the greatest common divisor

(gcd) of $\frac{r+1}{2}$ and $\frac{c+2}{2}$

## Arbitrariness

This feature of human language entails five aspects. First, the words of a language are generally recognized to represent a nexus between a group of sounds or signs that give the words their forms and the meanings, which the sounds are said to represent. Second, evidence of *arbitrariness* can be observed in the fact that the inner core of an object can be given a name. This evidence of arbitrariness can also be observed in cross-linguistic comparisons when words with the same meanings usually have different forms in different languages, and similar forms usually express different meanings. Third, *onomatopoeia* (i.e. words that imitate natural sounds or possess meanings that are associated with such sounds of nature) provides evidence of arbitrariness being the norm in languages, at least in terms of the connection between the forms of words and their meanings. Fourth, a counterexample of arbitrariness is *sound symbolism*: i.e. certain sounds occur in words not because they directly imitate some sounds but because they are evocative of particular meanings. Fifth, *non-arbitrariness* or *iconicity* plays a minimal role in language (Bergmann et al., 2007:18-21).

In her exploration of the classifications of the Niger-Congo languages, Gwenyth Lafleur, via Cynthia Hallen (1998-1999), points out that, when Malcolm Guthrie (1948) would find it difficult to employ the methods of dialect geographers to classify the Bantu languages, he would modify the approach by recognizing that the arbitrariness feature is vital to the method. According to Lafleur, Guthrie would begin with one language and move outwards, grouping with the language all the adjacent languages that possess similar characteristics. When Guthrie comes to a point where he has moved into another group, he would start the process over from the center.

Ellen Contini-Morava (1983) shows that while some rules of Kinyarwanda (a Bantu language of Central Africa) pronoun usage are motivated by arbitrariness, other rules are influenced by extra-linguistic factors. She argues that arbitrariness in language must therefore be perceived as a scale, given her findings on the order and interpretation of pronominal object prefixes in Kinyarwanda. She discovers that some pronoun sequences in the language are acceptable while others are not. Of those pronoun sequences that are acceptable, some provide information about the respective participants' roles of the pronouns' referents while others are ambiguous as to the roles. She therefore suggests the following:

> …both the restrictions of ordering possibilities and the interpretations of acceptable sequences can be explained in terms of two alternate strategies for the ranking of participants in a reported event: ranking according to degree of emphatic distance from ego (egocentricity) and ranking according to degree of potency in bringing about the event (contribution). Ambiguous pronoun sequences are cases where there is a potential conflict between these ranking strategies; unacceptable sequences violate the ranking according to egocentricity (Contini-Morava, 1983:425).

In their study of the theoretical cognitive interpretations of the Zulu noun class system, Britta Zawada and Mtholeni Ngcobo (2008) test the prevailing postulate that the order in which the noun class system of the Bantu languages of Southern Africa has developed suggests a morphologically arbitrary system, without any conceptual or semantic underpinning or purpose. They find that the Zulu noun class system is grounded on conceptual notions such as case of acquisition, frequency, and prototypicality.

In his *African Fractals: Modern Computing and Indigenous Designs*, Ron Eglash tells us that the idea of a spectrum progressing from more orderly to less orderly is quite evident in certain African material designs, which tend to reveal periodicity and aperiodicity—often moving from order to disorder—along the other. Similar geometric visualizations of this type of spectrum are employed in computer science (Eglash, 1999:172-173). Eglash cites the example of the random music of the Birom, a form of musical instrument indigenous to Nigeria that has something of a white noise distribution of sounds, as follows:

> …a flute ensemble designed to allow each musician to express individual feelings through whatever idiosyncratic noise (even silence) he or she chooses, resulting in an indeterminate process in which the sounds produced by the players are not obstructed by a conscious attempt to organize the rhythms and harmony (Eglash, 1999:174; partly quoting Akpabot, 1975:46).

The onomatopoeia of Birom, mimicking Eglash's use of the Rössler attractor as feedback (1999:169), can be represented in the following fractal mathematical expression:

Musical expression = Z

Total system: $x' = -(y + z)$

$$y' = x + 0.15y$$
$$z' = 0.2 + z(x - 10)$$

## Discreteness

*Discreteness* refers to that feature of language that makes it possible for humans to put together discrete units to make larger communicative units. This is imperative because every language has a limited number of sounds, between ten and 100. The sounds themselves for the most part lack meanings until they are combined together. The fact that humans can develop a large number of meaningful elements (words) from a few meaningless units (sounds) is known as *duality of patterning* (Bergmann et al., 2007:22).

In her article, “The Nature of Yoruba Intonation: A New Experimental Study” (2005), Eunice Fajobi discovers that when Yoruba tones are decontextualized for pedagogic reasons, they manifest discreteness. Fajobi therefore suggests that Yoruba merits the description of a register tone language.

According to Mirriam Nosiphiwo Ganiso (2012), South African Sign Language is a fully-fledged human language which possesses all the properties of a natural human language, including discreteness. This is mainly because, as she correctly points out, face-to-face interaction is a major requirement for communication in sign language.

And as Carlos Gussenhoven (1999) warns us, if “floating” tones of the sort that occur in African languages to mark clause structure and which are placed into the tonal string at the beginning of the sub-clause are to be excluded from intonational tones, then it behooves us to restrict such tones to those that either associate with metrically strong positions (“central tones”) or are used to mark off prosodic constituents (“boundary tones” or “peripheral tones”). He cites the example of some Nilo-Saharan languages that have breathy voice and laryngealized voice.

Eglash writes about discrete self-organizing in the Owari board game played throughout Africa in many different versions variously called *ayo, bao, giuthi, lela, mancala, omweso, owari, tei,* and *songo,* among many other names. Eglash describes the way the game is played as follows:

> The game is played by scooping pebble or seed counters from one cup, and placing one of those counters into each cup, starting with the cup to the right of the scoop. The goal is to have the last counter land in a cup that has only one or two counters already in it, which allows the player to capture these counters. In the Ghanaian game of owari, players are known for utilizing a series of moves they call a “marching group.” …if the number of counters in a series of cups each decrease by one (e.g., 4-3-2-1), the entire pattern can be replicated with a right-shift by scooping from the largest cup, and…if the pattern is left uninterrupted it can propagate in this way as far as needed for a winning move (1999:101).

The following is an example of a “marching group” strategy of 13 iterations provided by Eglash (1999:106):

3421→523→43111→4222→3331→442→5311→42211→3322→433→4411→4552→
3321→4321

Eglash notes that despite its simplicity, this concept of self-replicating pattern undergirds some sophisticated mathematical concepts. He also points out that John von Neumann, a pivotal contributor to the development of the modern digital computer, also birthed a mathematical theory of self-organizing systems, initially based on self-reproducing physical robots (Eglash, 1999:101).

## Displacement

What is termed *displacement* refers to the fact that speakers of a language can communicate about things, actions, ideas, etc. that are not spatially present while they are communicating (Bergmann, 2007:22). For example, people can talk about Allah (SWT) without actually seeing Him. They can talk about Prophet Muhammad (PBUH) or the Day of Judgment.

In his work on paradigmatic displacement, Russel Schuh proposes two aspects in which the feature is most common in West Chadic languages: (1) “analogical leveling of original distinctions on the model of one or more members of the paradigm” and (2) “to level variation at its original position while shifting it to some other point” (1980:355). He provides two illustrations—(1) Ngamo subject pronoun tone and (2) Bele object pronoun vowels—to support his propositions. He argues, however, that two factors are necessary for the paradigmatic displacement. The first factor is that there should be a site to which the displacement distinction is to be moved. The second factor is that the displaced distinction should not disrupt original distinctions to the point where communicative efficiency is hampered (1980:355-356).

Maarten Mous in his article titled “Loss of Linguistic Diversity in Africa” (2003) makes two important points about displacement. The first point is that displaced Bantu languages such as Ngoni from South Africa to Tanzania and the Bantu Mushungulu from Tanzania to Somalia reveal insights into general language adaptation, and that the mixed Swahili varieties along the Monzambique coast also show a loss of a special contact situation. The second point is that due to the prolonged war in their region and the fact that none of them had more than a few thousand speakers, Koman, a group of several languages in the Sudan-Ethiopia border, encountered displacement.

Oliver Bourderionnet employs the idea of displacement to investigate the work and situations of two francophone singer-songwriters from Africa: Tiken Jah Fakoly and Corneille. His objective is to provide answers to questions about language, identity, and the social role of the African pop artist in France and other francophone countries. Bourderionnet finds that the two artists “embody the mediation between publics from two continents and the global music industry in a particular geopolitical context. They also belong to a generation of artists whose productions signal a shift in French popular music representations of Africa and Africans. Discussing these artists’ choice to sing in French will also allow us to reflect on the position of the French language African artist in the English-dominated world of pop music” (2008:14).

In her study of the contention between Kiswahili and minority languages in Tanzania, Samantha Ross Hepworth (2008) focuses on one district of the country to demonstrate that even though Kiswahili has made a significant domain shift as it leaks from the public or formal domain into the vernacular, minority languages have been able to hold their own in at least the area of plant identification, knowledge and practices. The reason for the minority languages holding on to this ethnobotanical knowledge hinges upon the fact that local wild plants are basic sources to the speakers' livelihoods, as the plants are a source of food and medicine.

Robert Williams and Jade Comfort (2007) examine the displacement feature in their study of the Sudanese language Ghulfan among its speakers in the refugee community in Cairo, Egypt. They delineate two major findings. The first finding is that among the Ghulfan refugees, tone leveling is a possible result of urbanization. The second finding is that the younger speakers of the language living outside the Nubia Mountains can no longer understand other dialects in the group (Ghulfan, Tagle, and Dilling), although their elders still can.

In his discussion of the "Square Root, So-Called Pythagoreum Theorem, and Irrational Numbers," which appears in his *Civilization or Barbarism*, Diop shows how ancient Egyptians rigorously extracted the square root, even of the most complicated whole or fractional numbers. As he puts it (Diop, 1981/1991:258),

> The term that served to designate the square root in the Pharaonic language is significant in that respect: the right angle of a square, *knbt*; "to make the angle" = to extract the square root. Now the Egyptians defined a fundamental unit of length called "double remen," which is equal to the diagonal of a square of little side $a$ = one cubit (royal); in other words, if $d$ is that diagonal, then one necessarily has, by definition of the length itself, "double remen,"

$d = a\sqrt{2} = (\sqrt{2} \text{ X } 20.6) = 29.1325$ inches.
The royal cubit = 20.6 inches
The remen = $\frac{d}{2} = \frac{\sqrt{2}}{2}a = 14.6$ inches

Diop adds that since the Egyptians knew how to extract the surface of a triangle, they therefore wrote the following equation, followed by the extraction of the square root (1981/1991:260):

$S = \frac{1}{2}a^2 = \frac{1}{4}d^2 \rightarrow a^2 = \frac{1}{2}d^2 \rightarrow 2a^2 = d^2$
whence: $a\sqrt{2} = d = a$ double remen.

Appropriately, Diop makes the following assertion:

> This definition of the "double remen" by itself, and its mathematical implications, clearly show that Pythagoras was neither the inventor of irrational numbers (incommensurability of the diagonal and of the side of the square) nor of the theorem that bears his name: he took all these elements from Egypt where he had been, as reported by his biographers (cf. Jamblichus), a pupil of the priests for twenty-two years (1981/1991:260).

The preceding evidence clearly shows that Africans have been communicate about things, actions, ideas, etc. that are not spatially present while they are communicating for thousands of years.

## Productivity

*Productivity*, which augments *discreteness*, is about a language's capability to develop fresh meanings from discrete units. *Productivity* contrasts with *discreteness* in that the latter feature only requires re-combinable units in a fixed set of ways in which they could be combined while the former has no fixed set of ways in which units can be combined. Language *productivity* therefore makes it possible for humans to develop and comprehend any number of innovative statements that they may have never heard/read before, allowing them to express postulates that may never have been proffered before (Bergmann et al., 2007:22).

In his study of "low tone raising" (LTR) in the Hausa language, Schuh (1989) informs us that words in which the vowels are now long should be subject to LTR when it is a productive rule. He adds that a large proportion of these words was borrowed long before lengthening and determiners stopped being common and productive surface occurrences.

In another study titled "Palatalization in West Chadic" (2002), Schuh shows that Biu-Mandara languages display a type of morphological palatalization that is systemic and productive. He also reveals that Duwai and Ngizim have a productive rule that changes alveolar consonants other than lateral fricatives, *l* and *r*, to their palatal counterparts when followed by an *–i*. He further demonstrates that the process of palatalization of alveolars and consonants with palatal coarticulation is virtually 100% productive in Hausa.

Schuh in one more study, “The Locus of Pluractional Reduplication in West Chadic” (2002), reveals that in Hausa and a number of other Chadic languages, reduplication of a root initial syllable is the productive method of forming pluractional verbs. He notes that the productive pattern of verbal pluractional formation in Hausa is the doubling of the fist consonant/vowel/consonant (CVC) of a verb stem, with certain phonological processes that usually modify the final consonant of the initial syllable and/or its vowel. He adds that the only fully productive type of reduplication in the Bole language is the initial consonant/vowel (CV) of the verb.

We learn from Gideon Omachonu and David Abraham that in Igala, a West Benue-Congo language spoken in north central Nigeria, compounding is a highly productive word formation process in terms of varieties/forms and functions. They posit that semantic criteria above phonological and syntactic considerations are favorable when defining “compoundhood” and distinguishing compound words in Igala. As they put it,

> Apart from noun + noun compounds, other compound types such as synthetic and verbal compounds which could equally be accounted for using semantic criteria have been attested in Igala. Even though endocentric nominal compounds in Igala generally obey Left-Hand head rule (N1 as head), synthetic compounds in Igala, it has been observed, exhibit the possibility for either Left-Hand head position or Right-Hand head position. However, the Right-Hand head rule…may not be very productive in the language. It is adjudged as an exception rather than the rule. Lastly, in addition to the general function of lexical expansion through creation of new lexical categories or lexemes, compounding has been used copiously in naming concepts, particularly foreign institutions, ideas, items or objects and concepts that were hitherto non-existent in Igala (2012:201).

A good example of African mathematical productivity is Problem 79 in the *Ahmes Papyrus* which deals with a geometric progression of ratio seven and 3,500 years later was appropriated for the Mother Goose rhyme in English. Diop translates Problem 79, called the problem on “the inventory of *goods* contained in a house,” from hieroglyphics as follows: “there are seven houses; in each house there are seven cats, each cat kills seven mice; each mouse had eaten seven grains; each grain would have produced seven *hekat*. What is the sum of all the enumerated elements? (What is the total of all these things?)” Diop adds that the reasoning of the scribe yields the same numerical result as the application of the formula for modern algebra that yields the following sum of a geometric progression (1981/1991:270):

$$S = a\,\frac{r^n - 1}{r - 1} = 7 \text{ X } \frac{16{,}807 - 1}{7 - 1} = 7 \text{ X } \frac{16{,}806}{6} = 7 \text{ X } 2{,}801 = 19{,}607$$ [45]

As Beatrice Lumpkin points out, 3,500 years later, this geometric progression appeared as the Mother Goose rhyme in English as follows (1986:103):

> "As I was going to St. Tues,
> I met a man with seven wives.
> Each wife had seven cats…"

## The Role for Mathematicians in the African Renaissance

As stated earlier, mathematicians did play a significant role in the development of African societies during antiquity and that they are sorely needed now to help fulfill the African Renaissance. In this section, I suggest two ways mathematicians can work with experts in other disciplines to do so: (1) Mathematics of Sustainability and (2) Mathematics of Tipping Point. These are discussed in the following two subsections.

### Mathematics of Sustainability

To assert that African countries, like those on other continents, are quite ambitious when it comes to launching development projects is hardly a matter of dispute. The constant challenge, however, has been to ensure the Sustainability of many of these projects. The challenge prompted, to the best of my knowledge, two high-level meetings and two scholarly books to address it. The first meeting was convened by the United Nations on November 21, 2011 at which the organization's Deputy Secretary-General Asha-Rose Migiro emphasized the point that sustainable development is critical for addressing the economic, social and environmental challenges in Africa (United Nations News Centre, 2011). The second meeting was held in Cape Town, South Africa and brought together scientists and lawmakers from across Arica to discuss ways to promote sustainable development in Africa South of the Sahara (Voice of America, 2012). The first book titled *Sustainable Development in Africa* (2005) comprises papers from various contributors suggesting tools from various disciplines to examine factors limiting sustainable development in Africa and making recommendations. The second book, *Chemistry for Sustainable Development in Africa* (2013) entails a series of papers dealing with current chemical research in Africa focusing on environmental chemistry, renewable energies, health and human well-being, food and nutrition, and bioprospecting and commercial development.

The major shortcoming with all of these efforts from an African-centered perspective is that they offer no encompassing or indigenous African scientific approach to address the challenge of Sustainability in Africa. An answer to the challenge is Mathematics of Sustainability. Before discussing this aspect, it makes sense to first answer the following poignant question: What is all this talk about Sustainability?

Each year, we use more and more natural resources to maintain our way of life. Some of these resources are renewable and will never run out while others are fading quickly. Natural gas is slowly fading and efforts are being made to harness other energy sources such as solar, wind and, of course, electricity. Houses are being powered by solar panels, cars by electricity, and businesses by wind. These efforts help to keep our environment safe while also giving us the energy we need to survive. These new sources of energy do not only help us personally and environmentally, but also economically. These new ventures generate new businesses, and they can be quite profitable as well. Governments, different organizations, and numerous corporations are making an effort to provide a sustainable way of life, making the concept of *Sustainability* quite significant right now in science. But, what exactly is Sustainability? How do we use it? What purpose does it serve? According to the United States Environmental Protection Agency (EPA),

> Sustainability is based on a simple principle: Everything that we need for our survival and well-being depends, either directly or indirectly, on our natural environment. Sustainability creates and maintains the conditions under which humans and nature can exist in productive harmony, that permit fulfilling the social, economic, and other requirements of present and future generations (EPA, 2013).

For North Carolina State University (NCSU), the "concept of sustainability centers on a balance of society, economy and environment for current and future health. Responsible resource management in all three areas ensures that future generations will have the resources to survive and thrive" (NCSU, 2013). The NCSU definition adds the economy, while the EPA definition just deals with society and nature living in harmony. Thus, the NCSU definition is more comprehensive by providing us a nexus among society, nature, and economy.

With the broader definition, the fields in which the concept of Sustainability is being used can now be discussed. Just looking at the definition, it would seem that Sustainability can be used in almost any field, since it deals with society, nature, and the economy. But for the sake of this essay, let us look at the available research. Steve Cohen, director of the Master of Public Administration program in Environmental Studies at Columbia University, has found that there has been much growth in the field of Environmental Studies. As he notes,

> One of the most encouraging trends I’ve seen in recent years has been the growth of environmental studies programs in many American Universities. For the past decade I've directed an MPA program in Environmental Science and Policy at Columbia University's School of International and Public Affairs and Earth Institute. Today over 500 graduates of that program are working as environmental professionals all over the world (Cohen, 2012).

Cohen would go on to discuss how the school later launched a program in Sustainability Management and also how many other schools are developing Sustainability programs.

Kate Galbraith wrote an article for the *New York Times* titled “Sustainability Field Blooms on Campus” (2009), describing how schools are ramping up their Sustainability programs to meet the demand in society. Galbraith also discussed how many older individuals are going back to school for jobs in Sustainability fields. As she put it,

> Mr. Gressens’s trajectory will sound familiar at educational institutions across the country, whose continuing education arms have seen influx of students interested in the relatively new field of sustainability. At Harvard’s extension school, enrollment in environmental courses has soared by more than 70 percent in two years, according to the university, which has responded with new offerings in fast-changing fields like carbon neutrality and environmental economics (Galbraith, 2009).

Obviously, these institutions realize that this is a fast-growing market and are boosting their programs to meet the needs of society.

But still, what industries are using Sustainability the most? This question leads to an article by Helen Coster in the *Forbes* magazine which listed the top 100 mot sustainable companies. The list, which includes companies such as General Electric, H&M, Nokia, and Vodafone, shows that there exists a wide range of industries that use Sustainability. With this concept’s popularity and the growing threat of damage to our world, more industries and academic institutions are taking Sustainability more seriously and really putting their thoughts and money behind how to become more sustainable. As Cohen states,

> The challenges of global sustainability are also starting to influence higher education. The liberal arts have begun to reemphasize science, including earth sciences, ecology and environmental biology. The physics and finance of energy production and consumption are of obvious and increasing importance. My own academic field of organizational management is beginning to study the growing importance of the earth's resources in determining organizational effectiveness (Cohen, 2012).

So Sustainability is being used in almost every field now and everyone has a feeling of responsibility to society and the earth.

Along with the numerous fields in which Sustainability is being used, it is also being employed to study different aspects of life. In essence, Sustainability has an almost limitless potential for studying anything: the way we live, the way we do business, the way we go to school, and even the way we deal with other nations. Sustainability is used to study how we impact society, the environment, and the economy, which is mostly how we live in general. In short, Sustainability allows us to study the aspects of the way we live our lives.

A number of mathematical models have been used or suggested for measuring Sustainability. The following is an inventory of the works in the chronological order in which they were published:

(a) Amir Abbas Rassafi and Manouchehr Vaziri (2003) use pairwise correlation to examine Sustainability in 52 African countries.

(b) The Global Community WebNet (2004) suggests the use of the Gross Environmental Sustainable Development Index (GESDI), which quantitatively describes quality indicators rather than merely measuring different variables by focusing on the interactions between four major quality systems: (1) people, (2) economic development, (3) environment, and (4) availability of resources.

(c) Marion Hersh (2006) employs state space system representations, mental models, systems methodologies, optimization, mathematical decision making, multi-criteria problems, multi-criteria decision support methods, and fuzzy set operations to develop models for sustainable development.

(d) System Analysis Decisions (2009) utilizes a gauging matrix mathematical model to measure sustainable development, which is characterized by two main constituents: (1) security of population (Isec) and (2) quality of their life (Iql). The generalized sustainable development measure is then presented by a quaternary (Q) with an imaginary scalar part j(Isec), which describes the security of people, and a real vector part (Iqf), which describes quality of life in the space with three dimensions: (1) economic (Iec), (2) ecological (Ie), and (3) social-institutional (Is).

(e) Jason Phillips (2010) applies ideas of coupled environment-human systems to develop a mathematical model of sustainable development.

(f) Kalu A. Ugwa and A. Agwu (2012) tap difference equations, ordinary differential equations, partial differential equations, optimization modeling, simulation modeling, and function fitting data modeling to develop a mathematical tool for sustainable development in Nigeria.

(g) George Assaf et al. (2013) suggest life cycle analysis (LCA) that is useful for comparing the impacts of different activities in order to make hard choices for creating a climate-friendly sustainable energy future.

(h) Colin W. Clark (2013) implements the "logistic" model of population dynamics to measure sustainable resource management.

(i) Charles R. Hadlock (2013) uses differential equations, probability statistics, simulation, evolutionary game theory, and network theory to show how human society can be sustained in a rapidly changing world.

(j) Mark Alan Hughes and Elise Harrington (2013) show how the mathematics of Greenworks is comprised of baselines, absolute versus relative measurements, and interpolation.

(k) David Kung (2013) employs numbers, statistics, and functions to demonstrate the connection between social justice and Sustainability.

(l) Fred S. Roberts (2013) calls for the utilization of statistical and algorithmic methods of data analysis, advanced computational tools, and new cryptographic tools to aid us in making management and policy decisions about the electric power grid.

(m) The Sheward Partnership, LLC (2013) applies the LEED Rating System, which is the internationally-accepted benchmark for high performance green buildings, to calculate return on investment in sustainable design.

The problem with these models from an African-centered perspective is that a majority of them are not indigenous to the continent. They few that have their origins in Africa are not so identified, probably because the authors are oblivious to the fact.

Indeed, the notion of Sustainability was quite present in the minds of classic Egyptians. In Egyptian Hieroglyphic, the word *sānkhu* represented the word "peasant," "sustainer," or "vivifier" (Budge, 1978:645), meaning to give or bring life to, animate, to make more lively, intense, or striking enliven (http://www.thefreedictionary.com). In fact, in the funerary texts of later periods (c. 300 BC), Isis, the goddess of motherhood, magic and fertility (380-362 BC), due to her great power, was considered the mourner, protector and sustainer of the deceased in the afterlife. Isis' "protective and sustaining roles were extended to nobles and commoners and her power and appeal grew to the point that she eventually eclipsed Osiris (her brother and husband) himself and was venerated by virtually every ancient Egyptian" (http://egyptian-gods.99k.org/isis.html).

From Joyce Tyldesley, we also learn that Pharaoh Hatchepsut/Hatshepsut (throne name Maatkare, meaning Foremost of Noble Ladies, 1508-1458 BC), in her effort to ensure Sustainability in ancient Egypt, launched a number of bold initiatives. First, Hatchepsut increased the demand for scribes which led to an expansion of the education system. Second, the Pharaoh poured a lot of money into the building of monuments to ensure that the past is resuscitated and her name lives on forever, which in turn benefited a lot of artists and sculptors (Tyldesley, 1996:39-40). Third, she launched theological and technological advances to make sure that "New Kingdom Egypt remained tied to Middle and Old Kingdom Egypt by an unparalleled continuity of language, religion and artistic/architectural convention, and by the idiosyncratic Egyptian view of the world, and the position of Egypt, her people and her gods within the world, which had remained basically unchanged for over a thousand years" (Tyldesley, 1996:5-6). Fourth, she dreaded uncontrolled chaos, which was by definition a *maat*-less period, and therefore did everything in her power to avoid it by engaging in many international trade and diplomatic ventures (Tyldesley, 1996:8-10). Fifth, she advocated for large number of offspring, especially for the royal family which was plagued by the dearth of children, with sons particularly being in short supply and single daughters being the norm (Tyldesley, 1996:73-74).

In terms of the Mathematics of Sustainability in Africa, Diop teaches us plenty. First, the geometric progression of ratio seven, commonly called the problem on "the inventory of *goods* contained in a house," and the classical formula for the arithmetic progression, which I mentioned earlier, were invented in ancient Egypt. The mathematical series also invented included the following polygonal numbers (Diop, 1981/1991:270-272):

(a) trigonal or triangular numbers whose ratio, or difference of terms, is equal to 1

(b) tetragonal or square numbers such as 1, 4, 9, 16, 25, etc. whose difference in terms is 2, meaning that of the odd numbers 1, 3, 5, 7, 9, etc.

(c) *gnomons*, or successive rectangular belts, that allow one to obtain all squares from one unit square from the arithmetic series of odd numbers

(d) summing up of arithmetic progression: $S = \frac{N}{2}[2a + (n - 1)d]$[49]

In sum, these are all the elements that led to the "discoveries" of Pythagoras' theorem. This fact is echoed by Corinna Rossi when she points out that the first unambiguous evidence of the use of the theorem is a Demotic papyrus that dates back to the third century BC on which the numbers involved correspond to three triplets which might have been used as early as the Old Kingdom to construct some pyramids. Rossi observes that this truth is reflected in the insistence of the late Greek sources to link the 3-4-5 triangle to ancient Egypt (Rossi, 2007:64). She goes on to also cite the following most quoted passage from Plutach's *De hide et Osiride* ("His and Osiris") to prove the point:

> …the better and more divine nature consists of three elements—what is spiritually intelligible, the material and the element derived from these, which the Greeks call the cosmos. Plato is wont to call what is spiritually intelligible the form and the pattern of the father; and the material he calls the mother, the nurse, and the seat and place of creation, while the fruit of both he calls the offspring and creation. One might suppose that the Egyptians liken the nature of the universe especially to this supremely beautiful of the triangles which Plato also in the Republic seems to have used in devising his wedding figure. That triangle has a vertical of three units of length, a base of four, and an hypotenuse of five, which is equal, when squared, to the squares of the other two sides. The vertical should thus be likened to the male, the base to the female, and the hypotenuse to their offspring; and one should similarly view Osiris as the origin, Isis as the receptive element, and Horus as the perfect achievement. The number three is the first and perfect odd number; four is the square of the even number two; five is analogous partly to the father and partly to the mother, being made up of a triad and a dyad (Rossi, 2007:64).

Second, the following simple equations were invented by classic Egyptians (Diop, 1981/1991:272-274):

(a) the abstract and symbolic notation of the unknown quantity by assimilating the privileged number 1 for X

(b) the method of false supposition: A quantity (any) plus $^1/_7$ of it = 19; in modern algebraic form, A quantity X plus $^1/_7$ of it = 19; find X

(c) the simple equation of a more complex form: $X + \frac{2x}{3} - \frac{1}{3}(x + \frac{2x}{3}) = 10$

(d) the simple complex equation of $^2/_3$, plus $^1/_{10}$ of a number = 10; immediately the equation is written as follows: $(\frac{2}{3} + \frac{1}{10})X = 10 \quad X = 13\frac{1}{23}$

Third, classic Egyptians invented the following quadratic equations (Diop, 1981/1991:273-274):

$$\text{I} \quad \begin{cases} X^2 + Y^2 = 100 \\ 4X - 3Y = 0 \end{cases}$$
$$\text{II} \quad \begin{cases} X^2 + Y^2 = 400 \\ 4X - 3Y = 0 \end{cases}$$

The explicit wording of this system of simultaneous equations for the quadratic problem is the following: How to divide 100 into pairs, so that the square root of one of them is $^3/_4$ that of the other. Stated in modern symbols, the equation reads as follows:

$$X^2 + Y^2 = 100 \rightarrow Y = \frac{3}{4}X \rightarrow X^2 + \frac{9}{16}X^2 = 100$$

Fourth, the balance of quantities (*pesou*) was the invention of classic Egyptians. An example is that if the *pesou* of a loaf of bread is 12, then it means that this loaf of bread contains $^1/_{12}$ of a bushel. Also, the sacred right-angled triangle shows that some mathematical proportions had a divine essence. Furthermore, a table found from 2000 BC to 600 AD has a division of the number 2 by the odd numbers from 3 to 101. The table 2/*n* has no error. Even more, the Egyptian notation of fractions of 2200 BC was as follows (Diop, 1981/1991:274):

$$\frac{1}{17} \text{ of a silver talent} = 352 + \frac{1}{2} + \frac{1}{17} + \frac{1}{34} + \frac{1}{52} \text{ drachmas}$$

Finally, in the area of arithmetic, the following three observations by Diop are poignant (1981/1991: 276:276):

(1) The originality of Egyptian arithmetic is that it does not require any effort of memory. Multiplication and division are reduced to addition after a series of duplication. Only the multiplication by 2 needs to be known in order to easily carry out the most complex calculations.

(2) Operations on the fractions generally deal with fractions whose numerators equal a unit; however, the Egyptians knew and used also the following complimentary fractions: $^2/_3$ (frequently used); $^3/_4$, $^4/_5$, $^5/_6$ (less frequently used). Owing to this fact, the Egyptians had made a table of factorization of the fractions of type 2/*n*, including the fractions from $^2/_5$ to $^2/_{101}$.

(3) By the third millennium onward, the Egyptians had already invented decimal notation and discovered or portended the zero. The proportional divisions were known. They knew how to rigorously extract the square root of any number, even fractional ones. All these are evident in the examples dealt with by the Egyptian scribe Ahmes in the *Berlin Papyrus*.

And even more interesting is that, as Diop demonstrates, many Egyptian mathematical terms have survived in the Wolof language (1981/1991:226-228).

It should also be noted here that other scientific areas utilized to promote Sustainability in ancient Egypt included (a) Astronomy, encompassing the phases of the moon, the calendar, the orientation of the monuments, and the decans; (b) Medicine, (c) Chemistry, including the etymology of the word chemistry itself and the metallurgy of iron; and (d) Architecture, comprising the mathematical bases and the aesthetic canon of Egyptian art (for more on this, see Diop 1981/1991:278-307).

## Mathematics of Tipping Point

As Brett Cherry (2011), the Rensselaer Polytechnic Institute (2011) and Christian Kuehn et al. (2013) assert, the recent political revolutions that took place in Tunisia and Egypt are characteristic of Tipping Points, as they could result in vast social changes throughout North Africa and the Middle East, and affecting the rest of the world in the process. This conjecture raises at least two related questions:

(1) Why are some African political regimes suddenly changed, with no obvious trigger other than a slowly changing environment, while others that have the attributes of vulnerability thrive? (2) How can this phenomenon be predicted, so that mechanisms can be put in place to facilitate a political system's stability? A solution to this dilemma is *Mathematics of Tipping Point*.

When thinking of society and everything that happens on a day-to-day basis, when do things break or change? It seems as if life moves at a constant pace, consistently keeping us in a loop. At some point it feels like one small event pushes us over the edge and leads us to a major change that breaks us from this loop. This happens every day in our personal lives, societies, businesses, schools and, especially, our environment. What I am hinting at here is *Tipping Point*. This relatively new scientific concept seems to be popping up everywhere. But, what exactly is it? In my attempts to keep things simple, I will use a definition from dictionary.reference.com, which states that Tipping Point is "the point at which an issue, idea, product, etc. crosses a certain threshold and gains significant momentum, triggered by some minor factor or change; the point in a situation at which a minor development precipitates a crisis" (dictionary.reference.com). There is another definition of Tipping Point from the Mathematical Association of America (MAA), which states that

> The term "tipping point" describes the moment when a system suddenly changes state, with no obvious trigger other than a slowly changing environment. Tipping points are difficult to predict and difficult to reverse. Examples range from capsizing boats to fishery collapse; they include financial market crashes, the poverty trap, melting polar ice caps, shifts in ecosystems, and mood changes (MAA, 2013).

Like the example I used earlier, Tipping Point is when a situation like your life has a major change from a minor event in it. Since I used very simple definitions, it is not hard to see how Tipping Point is used across multiple disciplines or how it is applied in studying numerous phenomena. Malcolm Gladwell wrote a book titled *The Tipping Point: How Little Things Can Make a Big Difference* (2002), and in this book he discusses how he uses Tipping Point to describe how events take or have taken place in history as follows:

> The tipping point is the biography of an idea, and the idea is very simple. It is the best way to understand the emergence of fashion trends, the ebb and flow of crime waves, or, for that matter, the transformation of unknown books into bestsellers, or the rise of teenage smoking, or the phenomena of word of mouth, or any number of the other mysterious changes that mark everyday life is to think of them as epidemics. Ideas and products and messages and behaviors spread just like viruses do (Gladwell, 2002:3).

So Gladwell uses Tipping Point to examine all academic disciplines, making it a very useful concept for all fields of study. Gladwell also argues that no matter how different the disciplines may be, they can still be explained by using Tipping Point. For instance, Gladwell cites the rise of hush puppies and the fall of New York's crime rate as examples:

> The second distinguishing characteristic of these two examples is that in both cases little changes had big effects. All of the possible reasons for why New York's crime rate dropped are changes that happened at the margin; they were incremental changes. The crack trade leveled off. The population got a little older. The police force got a little better. Yet the effect was so dramatic. So too with Hush Puppies. How many kids are we talking about who began wearing the shoes in downtown Manhattan? Twenty? Fifty? One Hundred—at the most? Yet their actions seem to have single handedly started and international fashion trend (Gladwell, 2002:3).

This truism has propelled me to come to the conclusion that indeed Tipping Point can be used in all fields. A profitable question then is the following: Since Tipping Point can be applied in all fields, how exactly is it being used? To examine how the methodology is being used, I look at the field of Ecology because the environment is a big issue these days and is receiving a lot of attention from scholars, policymakers, and activists. In the Ecology field today, numerous studies are being done to determine how much harm is being done to the environment and how we can reverse these effects and live a more sustainable life. Tipping Point is used in these studies to determine what major change will occur from small occurrences. As Jeremy Hance points out,

> Even climate change, which some scientists might consider the ultimate tipping point, does not fit the bill, according to the paper. Impacts from climate change, while global, will not be uniform and hence not a tipping point as such; Local and regional ecosystems vary considerably in their responses to climate change, and their regime shifts are therefore likely to vary considerably across the terrestrial biosphere; from a planetary perspective, this diversity in ecosystem responses creates an essentially gradual pattern of change, without any identifiable tipping points (Hance, 2013).

Scientists are using the methodology of Tipping Point to determine whether or not we will have a great change in our ecosystem. In the article titled "Harnessing Math to Understand Tipping Points" (2013) Mary Lou Zeeman is reported to use mathematics to understand the Earth's climate. According to the article, "Co-director of the Mathematics and Climate Research Network which connects researchers deploying mathematics to better understand Earth's climate, Zeeman wants to empower those she teaches to seek solutions to today's environmental ills" (MAA, 2013:17).

Later in the article, Zeeman is noted to have given an example of how small events led to a much bigger change such as how the Earth moved out of its icy state millions of years ago as follows:

> Coming back to a question posed earlier in the lecture, Zeeman explained how Earth emerged from the snowball state it evidently occupied 600 million years ago. While volcanoes spewed greenhouse gases into the atmosphere, the planet's icy blanket prevented these gases from being removed via weathering of rocks or oceanic absorption of carbon. As greenhouse gases increased, the resilience of the stable snowball state decreased until, when it shrunk to nothing, the system tipped. A very hot Earth resulted: all the ice melted, rocks and oceans were exposed, and all those mechanisms for drawing carbon out of the atmosphere [were]…accessible again (MAA, 2013:17).

This example shows how Tipping Point is used to explain how our world is habitable right now. If Tipping Point can be used in something as large and serious as climate change and even how our world survived the ice age, then it can be employed to explain relatively minor phenomena such as things that occur in our daily lives or small changes in business.

Indeed, Tipping Point can be utilized to explore and learn quite a lot about various physical and socioeconomic phenomena, especially the aspect of how things change. How we move from small events to major events and how these events change our lives are pertinent to Tipping Point. It is used to study changes in crime rates and fashion trends like Gladwell shows us, or in climate change such as Zeeman shows us. By using Tipping Point and Sustainability together, we can find a better way to promote Africa's development. Also, by studying numerous fields across the board with these methodologies, we will have a better understanding of each other's disciplines, which will in turn give us a better understanding of each other's thoughts. The pace of our world is rapidly increasing with the advancement in technology. It is therefore imperative to have methodologies such as Tipping Point and Sustainability to help us understand our world and how cataclysmic events come about.

To determine whether or not accurate mathematical representations of physical or socioeconomic systems display Tipping Point behavior, bifurcation analysis is employed. As I recount in my book titled *African Mathematics: From Bones to Computers* (2012:121-125), while mathematicians are generally consistent in defining the term "first order differential equation," this is not the case for the term "Bifurcation." Instead, they tend to view Bifurcation as a description of certain phenomena. A common perception is that a system undergoes a bifurcation if and only if the global behavior of the system, which depends on a parameter, changes when the parameter varies. Consequently, a general and a theoretical definition of Bifurcation can be delineated.

Bifurcation can be generally defined as a differential equation system that undergoes a qualitative change in its orbit structure, as one or more parameters of the dynamical system are changed. A Bifurcation is said to occur when a small smooth change made to the parameter values, or the Bifurcation parameters, of a system causes a sudden qualitative or topological change in its short-term dynamical behavior. Bifurcations result in both continuous systems, described as ordinary differential equations (ODEs), delay differential equations (DDEs), and partial differential equations (PDEs); and discrete systems, described by maps.

Theoretically, Bifurcations can be defined as the mathematical study of changes in the qualitative or topological structure of the integral curves of a vector field, or the solutions of a differential equation. Essentially, a Bifurcation is said to occur at a parameter value where a number of solutions changes. In essence, Bifurcation Theory examines structurally unstable dynamical systems. Dynamic stability refers to perturbations in the phase space—i.e. the stability of fixed points and limit cycles; and structural stability refers to perturbations in the function space—i.e. the topological stability of orbit structures.

Bifurcations have been typologized in two ways. One typology, which is less technical, identifies three types of Bifurcations: (1) Subtle Bifurcations, whereby an attractor changes type; (2) Catastrophic Bifurcations, whereby attractors appear out of, or disappear into, the blue: and (3) Explosive Bifurcations, whereby attractors drastically change size. The other typology, which is more technical, identifies two types of Bifurcations: (1) Local Bifurcations that take place through changes in the local stability properties of the equilibria (i.e. fixed points), periodic orbits, or other invariant sets as parameters cross through critical thresholds; and (2) Global Bifurcations that take place when larger invariant sets of a system collide with one another, or with equilibria of a system. These Bifurcations cannot be detected purely by a stability analysis of the equilibria. Since the second typology is predominantly employed in Mathematics, a bit more discussion of their characteristics is in order.

A Local Bifurcation emerges when a parameter change causes the stability of an equilibrium to change. In continuous systems, this corresponds to the real part of an eigenvalue of an equilibrium passing through zero; in discrete systems, this corresponds to a fixed point having a Floquet Multiplier (relating to the class of solutions to linear differential equations) with modulus equal to one. In both cases, the equilibrium is non-hyperbolic at the Bifurcation Point (more on this later). The topological changes in the phase portrait of the system can be confined to arbitrarily small neighborhoods of the bifurcating fixed points by moving the Bifurcation parameter close to the Bifurcation Point—thus, the name "local."

Technically, the continuous dynamical system described by the ODE is considered as follows:

$$\dot{X} = f(x, \lambda) \quad f : \mathbb{R}^n \text{ x } \mathbb{R} \to \mathbb{R}^n$$

A Local Bifurcation emerges at ($x_0$,$\lambda_0$), if the Jacobian Matrix d$f_{x0,\lambda 0}$ has an eigenvalue with zero real part. If the eigenvalue is equal to zero, the Bifurcation is said to be a steady state Bifurcation; if the eigenvalue is non-zero, but purely imaginary, then it is said to be a Hopf Bifurcation (more on this later). For discrete dynamical systems, the following system is considered:

$$X_{n+1} = f(X_n, \lambda).$$

Consequently, a Local Bifurcation emerges at ($x_0$,$\lambda_0$), if the Jacobian Matrix d$f_{x0,\lambda 0}$ has an eigenvalue with modulus equal to one. If the eigenvalue is equal to one, the Bifurcation is either a Saddle-node—often called Fold Bifurcation when it appears in maps, Transcritical or Pitchfork Bifurcation. If the eigenvalue is equal to -1, it is a Period-doubling (or Flip) Bifurcation; otherwise, it is a Hopf Bifurcation. The following are examples of Local Bifurcations:

(a) Hopf Bifurcation—a Bifurcation in which a fixed point of a dynamical system loses stability as a pair of complex conjugate eigenvalues of the linearization around the fixed point cross the imaginary axis of the complex plane.

(b) Horse Saddle Bifurcation—a saddle point that is a minimax: i.e. a local minimum of maximum depending on the intersecting plane used.

(c) Monkey Saddle Bifurcation—an example of an immersion, it is a surface defined by the equation to which it belongs to the class of saddle surface, and its name derives from the observation that a saddle for a monkey requires three depressions: two for the legs and one for the tail.

(d) Neimark (Secondary Hopf) Bifurcation—a Bifurcation through which a system loses its stable period one operation.

(e) Period-doubling (Flip) Bifurcation—a Bifurcation in which the system switches to a new behavior with twice the period of the original system.

(f) Pitchfork Bifurcation—a generic Bifurcation in which a symmetric solution changes its stability; it occurs generically in systems with symmetry.

(g) Saddle-node Bifurcation—a Bifurcation in which two fixed points of a dynamical system collide and annihilate each other.

(h) Transcritical Bifurcation—a Bifurcation characterized by an equilibrium having an eigenvalue whose real part passes through zero.

A Global Bifurcation emerges when “larger” invariant sets, such as periodic orbits, collide with equilibria. Such a collision leads to changes in the topology of the trajectories in the phase space which cannot be confined to a small neighborhood, as is the case with Local Bifurcations. The changes in topology extend out to an arbitrarily large distance—hence, the name “global.” The following are examples of Global Bifurcations:

(a) Blue Sky Catastrophe, when a limit cycle collides with a nonhyperbolic cycle.

(b) Global Saddle (Fold) Bifurcation, when a system is expressed in polar coordinates.

(c) Harmoclinic Bifurcation, when a limit cycle collides with a saddle point.

(d) Heteroclinic Bifurcation, when a limit cycle collides with two or more saddle points.

(e) Infinite-period Bifurcation, when a stable node and saddle point simultaneously occur on a limit cycle.

It should be noted that Global Bifurcations can also involve more complex sets such as chaotic attractors.

The codimension of a Bifurcation refers to the number of parameters which must be varied for the Bifurcation to emerge. This corresponds to the codimension of the parameter set for which the Bifurcation emerges within the full space of parameters. Saddle-node Bifurcations are the only generic Local Bifurcations which are really codimension-one; the others all have higher codimension. Nonetheless, Transcritical and Pitchfork Bifurcations are also often perceived to be codimension-one, because the normal forms can be written with only one parameter.

An example of a well-studied codimension-two Bifurcation is the Bogdanov-Takens bifurcation, which is characterized by the fact that for the particular parameter value the vector field has a singularity whose linearized field has a double zero eigenvalue, while the other eigenvalues have nonzero real part. The latter condition is imperative.

The Bifurcation Diagram is a very useful tool employed to illustrate Bifurcations. It helps to show the possible long-term values—equilibria or periodic orbits—of a system function of a Bifurcation parameter in a dynamical system. It is normal practice to show stable solutions with a solid line and unstable solutions with a dotted line.

Named after their pioneer, Mitchell Feigenbaum, the Feigenbaum Constants are two mathematical constants used to express ratios in a Bifurcation Diagram. The first Fegenbaum Constant is expressed as follows:

$$\delta = 4.669201609102990671853203 28\ldots$$

where sequence A006890 in the On-line Encyclopedia of Integer Sequences (OEIS) is the limiting ratio of each bifurcation interval to the next, or between the diameters of successive circles on the real axis of the Mandelbrot set. While Feigenbaum originally related this number to the Period-doubling Bifurcations in the logistic map, he later showed it to hold for all one-dimensional maps with a single quadratic maximum. As a result of this generality, every chaotic system that corresponds to this description will bifurcate at the same rate. This Feigenbaum's constant is often employed to predict when chaos will arise in such systems even before it does. The second Feigenbaum Constant (sequence A006891 in the OEIS),

$$\alpha = 2.502907875095892822839028 73218\ldots$$

is the ratio between the width of a tine and the width of one of its two subtines (with the exception of the tine closest to the fold).

Both of the preceding numbers are applicable to a large class of dynamical systems. They are believed to be transcendental, albeit that claim remains to be proven.

Finally, Catastrophe Theory (which is also a special case of more general Singularity Theory in Geometry) considers the special case in the study of dynamical systems where the long-run stable equilibrium can be identified with the minimum of a smooth, well-defined potential function called Lyapunov Function. Since small changes in certain parameters of a nonlinear system can cause equilibria to appear or disappear, or to change from attracting to repelling and vice versa, this can lead to large and sudden changes of the behavior of a system. Investigated in a larger parameter space, Catastrophe Theory reveals that such Bifurcation points tend to occur as part of well-defined qualitative geometrical structures. Other aspects of Catastrophe Theory include Elementary Catastrophes, the potential functions of one active variable (Fold Catastrophe, Cusp Catastrophe, Swallowtail Catastrophe, and Butterfly Catastrophe), potential functions of two active variables (Hyperbolic Umbilic Catastrophe, Elliptic Umbilic Catastrophe, and Parabolic Umbilic Catastrope) and Arnold's notation, which are beyond the scope of this paper.

As I also discuss in *African Mathematics: From Bones to Computers*, mathematical aspects of Bifurcations are ubiquitous in ancient Egyptian artefacts. Indeed, classic Egyptians were quite involved with symbolism. Thus, their artefacts were designed and aligned cosmologically. The cosmological, astronomical, astrophysical, astrological and mathematical abilities of classic Egyptians were highly organized and quite advanced, and they performed truly remarkable feats of urban planning and architecture (Bangura, 2012:125).

For example, in their work on Auric Time Scale, Sergey Smelyakov, Geoff Stray and Jan Wicherink (2006a & 2006b) assert that in terms of proportions, the scale is reflected in the artefacts of Egypt. According to them, the Auric Time Scale $\Gamma$ with the radix $\varphi$ not only describes the crucial epochs in the known history of the Earth and humanity, it also adequately reflects the spectrum of the basic periods of Nature and society, including the geological and Solar activity cycles and artefacts of Egypt. In essence, embedded in Egyptian artefacts are representations of those events that have their impact on worldly events.

In order to get a sense of the connection between Egyptian artefacts and Bifurcations, it makes sense to quote verbatim what Smelyakov and his colleagues mean by Auric Time Scale. According to them, "In the narrow sense, the Auric Time Scale (ATS) is the series G of the Golden section powers (infinite to both ends), which is accompanied by the series G* = 2G of its double values; for its unity, phi^0 = 1 the average Solar cycle length T0 = 11.07 year, or Tropical year is taken" (Smelyakov et al., 2006a:1). They add that "In the broad sense, the ATS is a theory suggesting that for both principal aspects of time (periods and chronology), these series, T and T*, describe the length of the bulk of the basic periods in nature and society, and the major part of the most important historical and natural events in the evolutionary context, respectively" (Smelyakov et al., 2006a:1). I examine two examples of early Egyptian artefacts with aspects of Bifurcation in detail in *African Mathematics: From Bones to Computers*: (1) ancient Egyptian calendars and (2) the pyramid texts of Set (Seth, Seti, Setekh, Setesh, Suty, Sutekh) of Nubet (Bangura, 2012:126-144).

## Conclusions and Recommendations

The preceding analysis provides ample evidence that African languages exhibit all nine design features that can facilitate the domestication of mathematics. It is therefore only fitting that Africa was the center of mathematics history for tens of thousands of years. But since European languages have been privileged over African languages in mathematics education in the continent, we must do at least two very important things, if we are to remedy the problems that were stated earlier in the introduction section of this paper so that Africa can benefit from the tremendous opportunities mathematics offers. First, schools should encourage the use of African languages in order to nurture and promote those languages. Second, more research on the connection between language and the learning and teaching of mathematics from a political point of view is necessary.

Thus, the essence of an African-centered approach to mathematics education is that it is imperative and urgent for Africans to be concerned about broader development and the linguistic-mathematical nexus as well as approaches to these phenomena that are undergirded by humanity or fellow feeling toward others. When African-centeredness is considered along with the idea of the socialization effects of developmental environments and the possibilities of a reinforcement of these notions and contexts, the implications for African development, linguistic and mathematical processes appear vital.

Although compassion, warmth, understanding, caring, sharing, humanness, etc. are underscored by all the major world orientations, African-centered thought serves as a *distinctly African rationale* for these ways of relating to others. African-centeredness gives a distinctly African meaning to, and a reason or motivation for, a positive attitude towards the other. In light of the calls for an African Renaissance, African-centeredness urges Africans to be true to their promotion of good governance, democracy, peaceful relations and conflict resolution, educational and other developmental aspirations.

We ought never to falsify the cultural reality (life, art, literature) which is the goal of African-centeredness. Thus, we would have to oppose all sorts of simplified or supposedly simplified approaches and stress instead the methods which will achieve the best possible access to real life, language and philosophy.

## References

Agbinya, Johnson Ilyeh. 2004. *Computer Board Games of Africa: Algorithms, Strategies and Rules*. Bellville, South Africa: Department of Computer Science, University of Western Cape.

Akpabot, Samuel. 1975. Random music of the Birom. *African Arts* 8, 2:46-47.

Assaf, George, Eduardo Porta, Ralf Bredel and Cornelius Roschanek. April 2013. Creating a climate-friendly sustainable energy future: The role of mathematics. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Ayuninjam, Funwi F. 1998. *A Reference Grammar of Mbili*. Lanham, MD: University Press of America.

Bangura, Abdul Karim. 2012. *African Mathematics: From Bones to Computers*. San Diego, CA: Cognella Press.

Bangura, Abdul Karim. 2011. *The African National Anthem "Nkosi Sikelel' iAfrika": A Pragmatic Linguistic Analysis*. San Diego, CA: Cognella Press.

Bangura, Abdul Karim. 2005. Ubuntugogy: An African educational paradigm that transcends pedagogy, andragogy, ergonagy and heutagogy. *Journal of Third World Studies* xxii, 2:13-54.

Bangura, Abdul Karim. 2001. Book review of Ron Eglash's *African Fractals: Modern Computing and Indigenous Design*. *Nexus Network Journal* 2, 4.

Bangura, Abdul Karim. 2000. Book review of George Gheverghese Joseph's *The Crest of the Peacock: No-European Roots of Mathematics 2nd ed. Nexus Network Journal* 3, 3.

Bangura, Abdul Karim. 2000. *Chaos Theory and African Fractals*. Washington, DC: The African Institution.

Bangura, Abdul Karim. 2000. Measurable effects of multilingualism in Africa. *International Journal of the Sociology of Language* 146:111-117.

Bangura, Abdul Karim. 1991. *Multilingualism and Diglossia in Sierra Leone*. Lawrenceville, VA: Brunswick Publishing Corporation.

Bergmann, Anouschka, Kathleen Curie Hall and Sharon Miriam Ross. 2007. *Language Files: Materials for an Introduction to Language and Linguistics*. Columbus, OH: The Ohio University Press.

Bourderionnet, Oliver. 2008. Displacement in French/Displacement of French: The Reggae and R'n'B of Tiken Jah Fakolay and Corneille. *Research in African Literatures* 39, 4:14-23.

Budge, E. A. Wallis. 1978. *An Egyptian Hieroglyphic Dictionary* vols. I & II. New York, NY: Dover Publications, Inc.

Cherry, Brett. February 15, 2011. Egypt's 'tipping point'? *RSS feed*. Retrieved on November 02, 2011 from http://tippingpointsproject.org/2011/02/15/egypts-tipping-point/

Chomsky, Noam and Morris Halle. 1968. *The Sound Pattern of English*. New York, NY: Harper and Row.

Clark, Colin W. April 2013. Mathematics and resource management. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Cohen, Steve. February 13, 2012. The growing field of sustainability. *Huffington Post*. Retrieved on November 03, 2013 from http://www.huffingtonpost.com/steven-cohen/the-growing-field-of-sust_b_1272831.html

Contini-Morava, Ellen. 1983. Ranking of participants in Kinyarwanda: The limitations of arbitrariness in language. *Anthropological Linguistics* 24, 4:425-435.

Coster, Helen. January 27, 2010. Ranking the world's most sustainable companies. *Forbes*. Retrieved on November 03, 2013 from http://www.forbes.com/2010/01/26/most-sustainable-companies-leadership-citizenship-100.html

Creissels, Denis. 2000. Typology. In B. Heine and D. Nurse, eds. *African Languages: An Introduction*. Cambridge, UK: Cambridge University Press.

Dahl, Øyvind. 1999. *Meanings in Madagascar: Cases of Intercultural Communication*. Westport, CT: Bergin and Garvey, an imprint of Greenwood Publishing Group, Inc.

DawaNet. 2013. *American Muslim History*. Retrieved on June 01, 2013 from http://www.dawanet.com/history/amermuslimhist.asp

Devlin, Keith. 2003. *Mathematics: The Science of Patterns: The Search for Order in Life, Mind and the Universe*. New York, NY: Henry Holt.

Dictionary.reference.org. 2013. Tipping point. Retrieved on November 03, 2013 from http://dictionary.reference.com/browse/tipping+point

Dinneen, Francis P. 1995. *General Linguistics*. Washington, DC: Georgetown University Press.

Diop, Cheikh Anta. 1987. *Precolonial Black Africa*. Chicago, IL : Lawrence Hill Books.

Diop, Cheikh Anta. 1981/1991. *Civilization or Barbarism: An Authentic Anthropology*. Paris, France: Présence Africaine (English translation by Yaa-Lengi Meema Ngemi and published by Lawrence Hill Books, Chicago, IL, 1991).

Djebbar, Ahmed. 1995. On mathematical activities in North Africa since the 9th Century. *AMUCHA Newsletter 15*. Retrieved on January 25, 2007 from http://www.math.buffalo.edu/mad/AMU/amu+chma_15.html#2

Eglash, Ron. 1999. *African Fractals: Modern Computing and Indigenous Design*. New Brunswick, NJ: Rutgers University Press.

Ehret, Christopher. 2000. Language and history. In B. Heine and D. Nurse, eds. *African Languages: An Introduction*. Cambridge, UK: Cambridge University Press.

Environmental Protection Agency (EPA). 2013. What is sustainability. Retrieved on November 03, 2013 from http://www.epa.gov/sustainability/basicinfo.htm

Fajobi, Eunice. 2005. The nature of Yoruba intonation: A new experimental study. In T. Falola and A. Genova, eds., *Yoruba Creativity: Fiction, Language, Life and Songs*. Trenton, NJ: Africa World Press.

Galbraith, Kate. August 19, 2009. Sustainability field booms on campus. *NY Times*. Retrieved on November 03, 2013 from http://www.nytimes.com/2009/08/20/education/20GREEN.html?_r=1&

Ganiso, Mirriam Nosiphiwo. 2012. Sign language in South Africa: Language planning and policy challenges. Master of Arts Thesis, School of Languages: African Language Studies, Rhodes University, Grahamstown, South Africa.

Gerdes, Paulus. 2009. *Sipatsi: Basketry and Geometry in the Tonga Culture of Inhambane*. Maputo, Mozambique: Centre for Mozambican Studies and Ethnoscience, Universidade Pedagógica.

Gerdes, Paulus. 1999. *Geometry from Africa: Mathematical and Educational Explorations*. Washington, DC: The Mathematical Association of America.

Gladwell, Malcolm. 2002. *The Tipping Point: How Little Things Can Make a Big Difference*. New York, NY: Little Brown and Company. January 2002.

Global Community WebNet Ltd. 2004. Measurement of sustainable development. Retrieved on May 03, 2013 from http://globalcommunitywebnet.com/globalcommunity/measurementofsd.htm

Gnaedinger, Franz. 2005. Very early calendars. Retrieved on July 22, 2008. Available at http:// http://www.seshat.ch/home/calendar.htm

Gurib-Fakim, Ameenah and Jacobus Nicolaas Eloff (eds.). 2013. *Chemistry for Sustainable Development in Africa.* Heidelberg, Germany: Berlin Springer.

Gussenhoven, Carlos. 1999. Discreteness and gradience in intonational contrasts. *Language and Speech* 42, 2-3:283-305.

Guthrie, Malcolm. 1948. *The Classification of the Bantu Languages*. London, UK: Oxford University Press.

Hadlock, Charles R. April 2013. Sustaining human society in a rapidly changing world. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Hance, Jeremy. March 06, 2013. Warnings of global ecological tipping points. *Environmental Network News*. Retrieved on November 03, 2013 from http://www.enn.com/sustainability/article/45684

Hayward, Richard J. 2000. Afroasiatic. In B. Heine and D. Nurse, eds. *African Languages: An Introduction*. Cambridge, UK: Cambridge University Press.

Hepworth, Samantha Ross. 2008. Absorption or displacement – necessary or gratuitous? The contention between Kiswahili and minority languages in Tanzania. Retrieved on June 07, 2013 from http://www.seameo.org/_ld2008/doucments/Presentation_document/MicrosoftWordAbsorptionOrdisplacement_BKKpaper_SamanthaRossHepworth.pdf

Hersh, Marion A. 2006. *Mathematical Modelling for Sustainable Development*. Berlin, Germany: Springer.

http://egyptian-gods.99k.org/isis.html

http://www.math.buffalo.edu/mad/Ancient-Africa

Hughes, Mark Alan and Elise Harrington. April 2013. Making greenworks count. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Hunter, Brad. n.d. Tipping points in social networks. Retrieved on November 03, 2013 from. http://www.stanford.edu/class/symbsys205/tipping_point.html

Joseph, George. Gherverghese. 1991/2000. *The Crest of the Peacock: Non-European Roots of Mathematics*. New York, NY: Penguin Books.

Kuehn, Christian, Erik A. Martens and Daniel M. Romero. July 31, 2013. Critical transitions in social network activity. Retrieved on November 02, 2013 from http://arxiv.org/pdf/1307.8250.pdf

Kung, David. April 2013. Mathematics, social justice & sustainability. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Lafleur, Gwenyth J. via Cynthia L. Hallen. 1998-1999. Exploring the Niger-Congo languages. Retrieved on June 01, 2013 from http://linguistics.byu.edu/classes/ling450ch/reports/niger-congo.html

Lumpkin, Beatrice. 1986. Africa in the mainstream of mathematics history. I. V. Sertima, ed. *Blacks in Science: Ancient and Modern*. New Brunswick, NJ: Transaction Books.

Mankiewicz, Richard. 2001. *The Story of Mathematics*. Princeton, NJ: Princeton University Press.

Mathematical Association of America. March 28, 2013. Harnessing math to understand tipping points. Retrieved on November 03, 2013from http://archive.is/zGOtm

Meli, Francis. 1988. *South Africa Belongs to Us: A History of the ANC*. Harare, Zimbabwe: Zimbabwe Publishing House (republished in 1989, London, UK: James Currey).

Montague, Richard. 1974. *Formal Philosophy* (Richmond H. Thomason, ed.). New Haven, CT: Yale University Press.

Mous, Maarten. 2003. Loss of linguistic diversity in Africa. Retrieved on June 07, 2013 from http://www.ddl.ish-lyon.cnrs.fr/Fulltext/philippson/Mous_LossLinguisticsDiversityAfrica.pdf

Nabudere, Dani Wadada. 2003. Towards the establishment of a Pan-African university: A strategic concept paper. *African Journal of Political Science* 8, 1:1-30.

North Carolina State University (NCSU). 2013. What is sustainability? *Sustainability at NC State*. Retrieved on November 03, 2013 from http://sustainability.ncsu.edu/about/what-is-sustainability

Omachonu, Gideon S. and David A. Abraham. 2012. Compounding in Igala. *Unizik Journal of Arts and Humanities* 13, 2:184-206.

Papakonstantinou, Joanna M. and Richard A. Tapia. 2013. Origin and evolution of the secant method in one dimension. *The American Mathematical Monthly* 120, 6:500-517).

Phillips, Jason. May 2010. A mathematical model of sustainable development using ideas of coupled environment-human systems. *The Pelican Web's Journal of Sustainable Development* 6, 5.

Pullum, Geoffrey K. and András Kornai. 2003. Mathematical linguistics. *The Oxford International Encyclopedia* 2nd ed., 17-20. Oxford, UK: Oxford University Press. Retrieved on June 08, 2013 from http://www.kornai.com/MatLing/matling3.pdf

Rassafi, Amir Abbas and Manouchehr Vaziri. Fall 2003. African sustainable transport by numbers. *Journal of Sustainable Development in* AfricaRetrieved on May 03, 2013 from http://www.jsd-africa.com/Jsda/Fall2003/articlepdf/ARC-African%20Sustainable%20Transport%20by%20number%20%28article%29.pdf

Rensselaer Polytechnic Institute. July 26, 2011. Minority rules: Scientists discover tipping point for the spread of ideas. *Science Daily*. Retrieved on November 02, 2013 from http://www.sciencedaily.com/releases/2011/07/110725190044.htm

Roberts, Fred S. April 2013. Energy as a contributor to human well-being: Electric power grids. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Rossi, Corinna. 2007. *Architecture and Mathematics in Ancient Egypt*. New York, NY: Cambridge University Press.

Schuh, Russel G. 2002. Palatalization in West Chadic. *UCLA Linguistics Papers*. Retrieved on June 07, 2013 from http://www.linguistics.ucla.edu/people/schuh/Papers/A78_2002_WC_palatalization.pdf

Schuh, Russel G. 2002. The locus of pluractional reduplication in West Chadic. *UCLA Linguistics Papers*. http://www.linguistics.ucla.edu/people/schuh/Papers/ms_2002_WC_redup.pdf

Schuh, Russel G. 1989. The reality of Hausa “low tone raising”: A response to Newman & Jaggar. *Studies in African Linguistics* 20, 3:257-262.

Schuh, Russel G. 1980. Paradigmatic displacement. *UCLA Linguistics Papers*. Retrieved on June 07, 2013 from
http://www.linguistics.ucla.edu/people/schuh/Papers/A32_1980_paradigmatic_displ.pdf

Setati, Mamokgethi. 2008. Access to mathematics versus access to the language of power: The struggle in multilingual mathematics classrooms. *South African Journal of Education* 28:103-116.

Setati, Mamokgethi. 2005a. Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education* 36, 5:447-466.

Setati, Mamokgethi. 2005b. Researching teaching and learning from “with” *or* “on” teachers to “with” *and* “on” teachers. *Perspectives in Education* 23, 1:91-101.

Setati, Mamokgethi. 2003. ‘Re’-presenting qualitative data from multilingual mathematics classrooms. *ZDM* (Zentralblatt für Didaktik der Mathematik)*: The International Journal on Mathematics Education* 35, 6:294-300.

Setati, Mamokgethi. 2002. Researching mathematics education and language in multilingual South Africa. *The Mathematics Educator* 12, 2:6-20.

Setati, Mamokgethi. 1998. Code-switching in a senior primary class of second-language mathematics learners. *For the Learning of Mathematics* 18, 1:34-40.

Smelyakov, S. V., G. Stray and J. Wicherink. 2006a. The Auric Time Scale. *Astrotheos*. Retrieved on August 9, 2008. Available at http://www.ASTROTHEOS.narod.ru

Smelyakov, S. V., G. Stray and J. Wicherink. 2006b. The last multi-turns of the spiral of time before it rolls up to appear in new reality. *Astrotheos*. Retrieved on August 9, 2008. Available at http://www.ASTROTHEOS.narod.ru

System Analysis & Decisions. 2009. Sustainable development: Gauging matrix mathematical model. Retrieved on May 03, 2013 from http://systemdecisions.com/research/sdgm-mathematical-model

*The Free Dictionary*. Available at http://www.thefreedictionary.com

*The Internet Encyclopedia of Science*. Lebombo bone. Retrieved on July 25, 2007 from http://www.daviddarling.info/encyclopedia/L/Lebombo_bone.html

The Sheward Partnership, LLC. April 2013. Calculating return on investment in sustainable design. Mathematics Awareness Month Homepage. Retrieved on February 27, 2013 from http://www.mathaware.org/index.html

Tyldesley, Joyce. 1996. *Hatchepsut: The Female Pharaoh*. New York, NY: Viking/Penguin Group.

Ugwa, Kalu A. and Agwu A. April 2012. Mathematical modeling as a tool for sustainable development in Nigeria. *International Journal of Academic Research in Progressive Education and Development* 1, 2:251-258.

Ukaga, Okechukwu and Osita George Afoaku. 2005. *Sustainable Development in Africa*. Trenton, NJ: Africa World Press.

United Nations News Centre. November 21, 2011. Sustainable development key to Africa's socio-economic challenges. Retrieved on October 24, 2013 from http://www.un.org/apps/news/story.asp/html/story.asp?NewsID=40463&Cr=sustainable+development&Cr1=#.UoCXYSfAZfQ

Voice of America. November 01, 2012. 'Future Earth' discusses sustainable development challenges for Africa. Retrieved on October 24, 2013 from http://www.voanews.com/content/experts-discuss-sustainable-development-for-africa/1537550.html

Watters, John R. 2000. Syntax. In B. Heine and D. Nurse, eds. *African Languages: An Introduction*. Cambridge, UK: Cambridge University Press.

Williams, Robert S. and Jade Comfort. 2007. Language documentation and description among refugee populations. In P. K. Austin, O. Bond and D. Nathan, eds. *Proceedings of Conference on Language Documentation and Linguistic Theory*. London, UK: School of Oriental and African Studies, University of London, UK, December 7-8:261-268.

Winters, Clyde-Ahmad. 1986. The ancient Manding script. In I. V. Sertima, ed. *Blacks in Science: Ancient and Modern*. New Brunswick, NJ: Transaction Books.

Wolff, Ekkehard. 2000. Language and society. In B. Heine and D. Nurse, eds. *African Languages: An Introduction*. Cambridge, UK: Cambridge University Press.

Zawada, Britta and Mtholeni N. Ngcobo. 2008. A cognitive and corpus-linguistic re-analysis of the acquisition of the Zulu noun class system. *Language Matters: Studies in the Languages of Africa* 39, 2:316-331.

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